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A kinematic model for calculating cup alignment error during total hip arthroplasty

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Abstract

Reduced range of motion, prosthetic impingement, and joint dislocation can all result from misalignment of the acetabular component (i.e. cup alignment) in patients undergoing total hip arthroplasty. Most methods for acetabular component alignment are designed to provide 45–50° abduction and 15–25° of operative anteversion (also known as flexion) with respect to the anterior pelvic plane coordinate system. Yet in most cases, this coordinate system is not assigned properly, due to differences in patient anatomy and improper positioning in the operating room. This misalignment can result in an error in the cup alignment, which can cause the above-mentioned consequences. This work presents a complete mathematical formulation for the analysis of the inaccuracies related to the anterior pelvic plane axes (APPA) definition and their effect on final cup orientation. We do this by introducing a method taken from Kinematics of Mechanisms, and by representing the errors in the APPA as three concurrent axes of rotation, followed by the version and abduction rotations which are defined relative to the previous rotations. We also present a sensitivity analysis of the results by introducing differential changes between sequential coordinate frames, which simulates the errors in the APPA and their effect on cup orientation. Finally, we demonstrate a computational method which provides corrected version and abduction angles to achieve the desired cup orientation, given that the actual measurement errors are known.

Keywords: Hip arthroplasty; Cup alignment error Kinematic model; Anterior pelvic plane axes; Pelvic biomechanics

1. Introduction

Alignment of the acetabular component (i.e. cup alignment) during total hip replacement is a crucial step for reducing the chances for joint dislocation (Nolan et al., 1975; Fackler and Poss, 1980; Khan et al., 1981; Woo and Morrey, 1982; Dorr et al., 1983; Kristiansen et al., 1985; Garcia-Cimberlo and Munuera, 1992), and impingement, in patients undergoing total hip artroplasty (THA) (Coventry et al., 1974; Lewinnek et al., 1978). Consequently, the determination of the optimal orientation of the acetabular components during THA has been the focus of numerous studies. A wide range of parameters is reported (Visser and Konings, 1981). Harris (1980) suggests 30° of abduction and 20° anteversion for cup orientation. Hakess suggests the abduction angle of 45° and an anteversion of 15±5° (Harkess, 1992), and Lewinnek recommends an abduction angle of 40±10° and anteversion of 15±5° (Lewinnek et al., 1978). However, most methods for acetabular components alignment are designed to provide 45–50° abduction and 15–25°of operative anteversion (also known as flexion) with respect to the anterior pelvic plane. Moreover, implant manufacturers usually provide mechanical guides that place the acetabular components at 45° and 20° abduction and operative version, respectively. However, these mechanical guides assume a fixed, predetermined, pelvic orientation, when in practice, it has been shown that the position of the acetabular component may vary

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depending on the pelvis orientation on the operating table (Schmalzried et al., 1994). McCollum and Gray (1990) claim that accurately aligning the pelvis with respect to the patient in the lateral decubitus is almost an impossible task. In their work, McCollum and Gray also state that pelvic malalignment could lead to improper cup alignment. The researchers also indicate that pelvis flexion, and soft tissue contractures can result in changes in native acetabular orientation from the apparent position on the operating table, and may lead to component malposition.

Schneider et al. (1982) suggested standardizing the position of the patient on the operating table by setting the position of the central X-ray beam and the anterior pelvic plane (APP) alignment to a standard value. In order to define the APP, one needs to determine the anterior superior iliac spines, and the pubic tubercles. Yet, while determining these anatomic landmarks, one can introduce errors which affect the final definition of the APP, resulting in improper cup alignment. There are three definitions of cup orientation commonly used in clinical practice, each resulting from a particular application (Fig. 1): radiographic, operative, and anatomic (Murray, 1993). However, these methods do not take into account error in the anterior pelvic plane axes definition, and how would these errors affect cup orientation. Also they do not provide a tool to determine what should the abduction version angles be in order to compensate for these errors and still accomplish desired cup orientation.

This work presents a complete mathematical formulation for the analysis of the inaccuracies related to the anterior pelvic plane axes (APPA) definition and their effect on final cup orientation. We do this by introducing a method taken from the kinematics of mechanisms, and by representing the errors in APPA (Fig. 2) as three concurrent axes of rotation, followed by the version and abduction rotations which are defined relative to the previous rotations. The transverse axis is defined as a line connecting the superior anterior iliac spine points. The anterior pelvic plane is then defined by the transverse axis and the mid point between the two pubis symphysis tubercles. The second axis of the coordinate system lies in that plane and is perpendicular to the transverse axis and the third axis is perpendicular to the APP. We also present a sensitivity

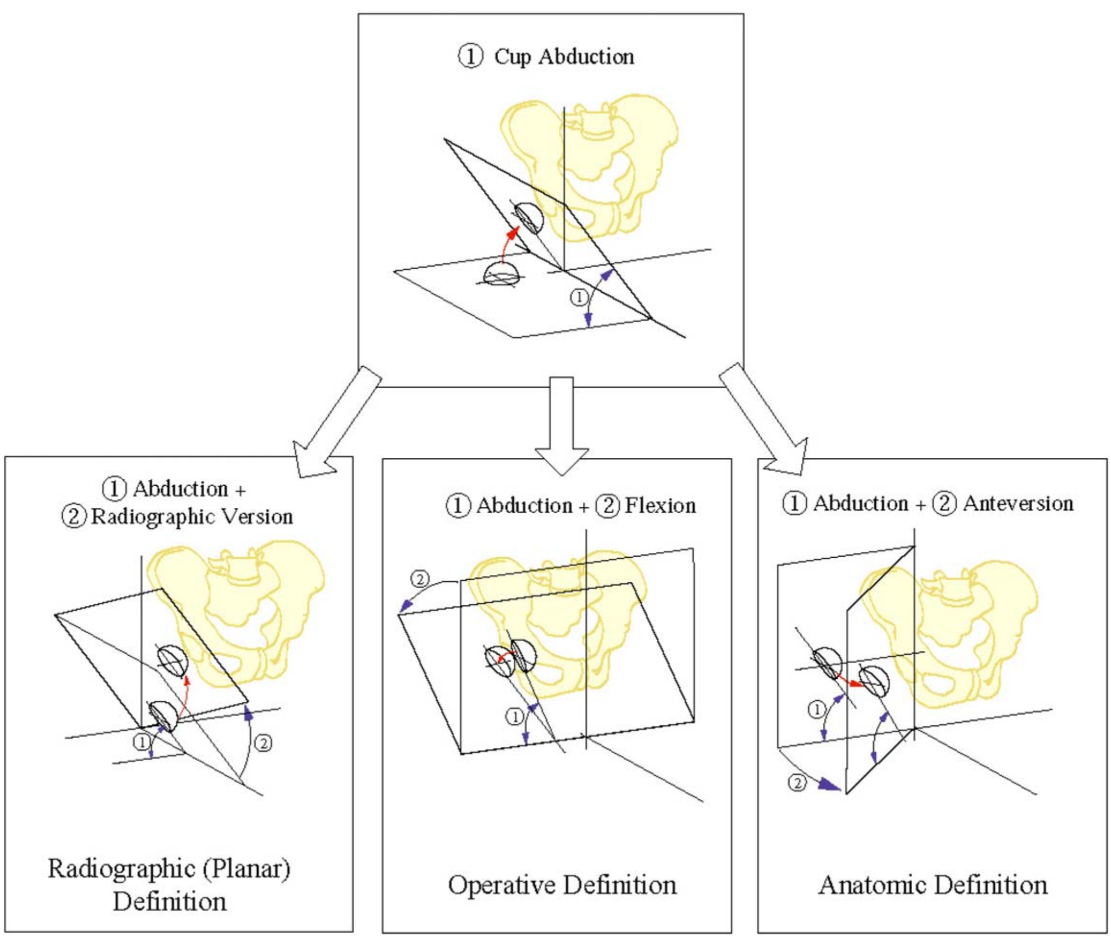

Fig. 1. Definition of cup orientation (Jaramaz et al., 1998).

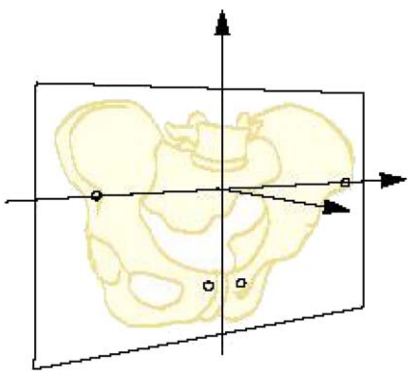

Fig. 2. Anterior pelvic plane axes (APPA).

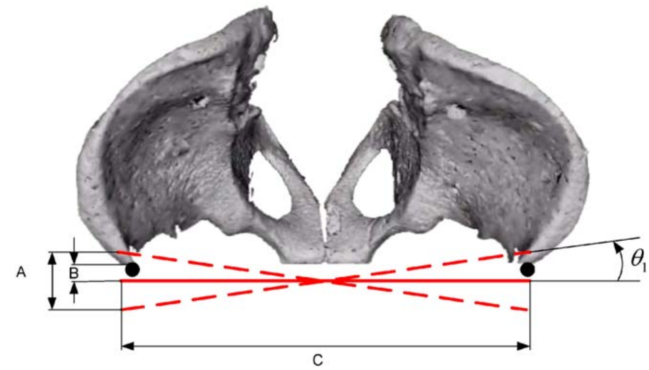

Fig. 3. Error in determining the anterior superior iliac spines, a bias error of B with a range of error A, resulting in anterior–posterior error version angle θ_1.

analysis of the results by introducing differential changes between sequential coordinate frames, which simulates the errors in the APPA, and their effect on cup orientation. Finally, we also demonstrate a computational method which provides corrected version and abduction angles to achieve the desired cup orientation, given that the actual measurement errors are known.

2. Materials and methods

2.1. Kinematics skeleton model of cup orientation and its errors

As was suggested by Lewinnek and coworkers (Lewinnek et al., 1978), in order to define the APP, one needs to determine the anterior superior iliac spines and the pubic tubercles. While determining these anatomic landmarks, one can introduce errors which affect the definition of the APPA orientations (Figs. 3–5). As was reported before, the nominal version and abduction angles of the cup are defined with respect to the rotated APP resulting from inaccurate localization. A kinematic skeleton model of the described system is presented in Fig. 6. As we do not care (in this case) about the location of the cup, the kinematic skeleton is composed of a set of revolute joints (one rotational degree of freedom each), relating the world coordinate reference frame on the operating table (O_0), to the cup coordinate system (O_5).

Referring to Fig. 6, the first three joints (J1–J3) represent the three rotational errors given (Figs. 3–5), while the last two joints (J4, J5), represent the nominal version and abduction of the cup, i.e. the intended cup orientations. By presenting cup alignment and resulting mesurements errors as a set of revolute joints aligned in a kinematic skeleton, we are able to apply methods taken from robotics and the kinematics of mechanisms in order to quantify the total error and the effect of each

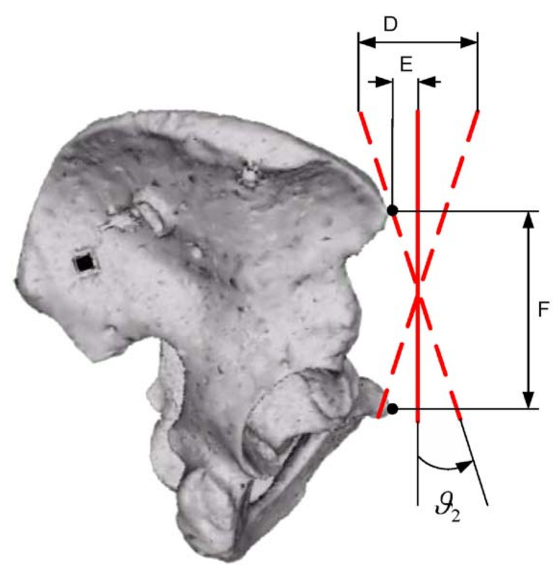

Fig. 4. Error in the determining the pubic tubercles and the anterior superior iliac spines, a bias error of E with a range of error D, resulting in flexion error angle θ_2.

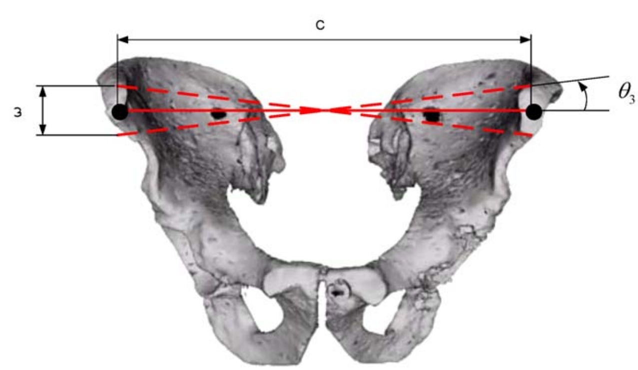

Fig. 5. Error in the determining anterior superior iliac spines, resulting in superior-inferior error angle θ_3.

of the kinematic parameters on the total resulting error. Following are the kinematic analysis tools which are used to investigate our system.

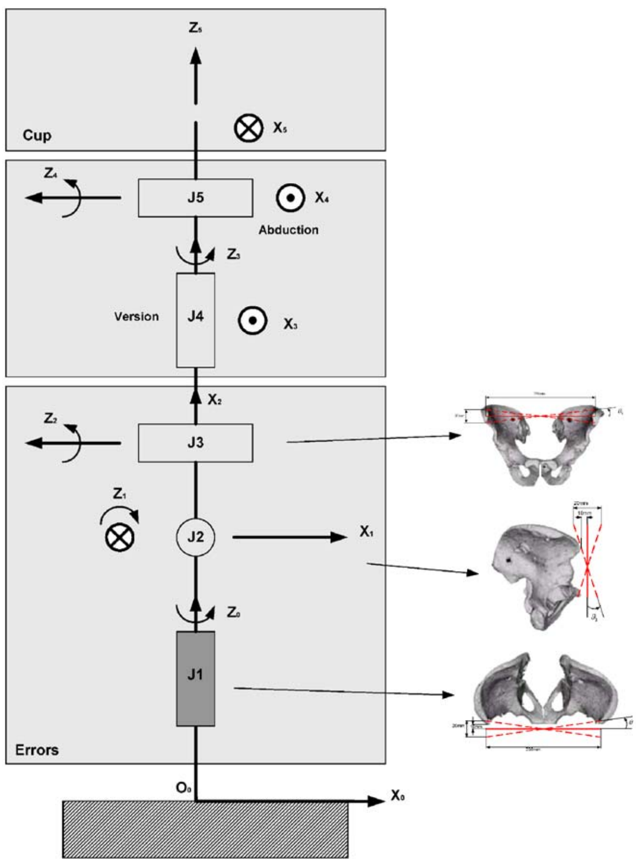

Fig. 6. A kinematic skeleton simulating cup alignment and errors.

2.2. Kinematics analysis tools

The relationship between two succesive coordinate frames, i and $i+1$, can be derived using the Denavit and Hartenberg method (Denavit and Hartenberg, 1955). In order to use their method, one has to define each of the four kinematic parameters of the ith joint. These parameters are: $\theta_i, r_i, l_i, \alpha_i$ (Fig. 7). Using these parameters, it is possible to define the transformation matrix between coordinate frame $i-1$, and i. This matrix, denoted by A_i, is given as

$$A_i = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha & l_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & l_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & r_i \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{1}$$

where $C\theta, S\theta$ stands for the cosine and sine function of an angle.

After defining $A_1, \ldots, A_N$ (for $i = N$, joint), one can define the transformation matrix, T_N, relating the position (location and orientation) of the N degrees of freedom (DOF) system in world coordinate frame as

$$T_N = A_1 {*} A_2 {*}, \ldots, A_N, \tag{2}$$

where $*$ represents matrix multiplication.

As one can observe from Eqs. (1) and (2), T_N depends on the $4N$ kinematic parameters $\theta_i, r_i, l_i, \alpha_i$; hence, errors in these parameters are reflected in T_N. We refer the readers to Paul (1981) and Wu (1984) to learn more about how to quantify these kinematics errors. Following is a short summary of the method presented in Wu (1984), which compute the error in T_N.

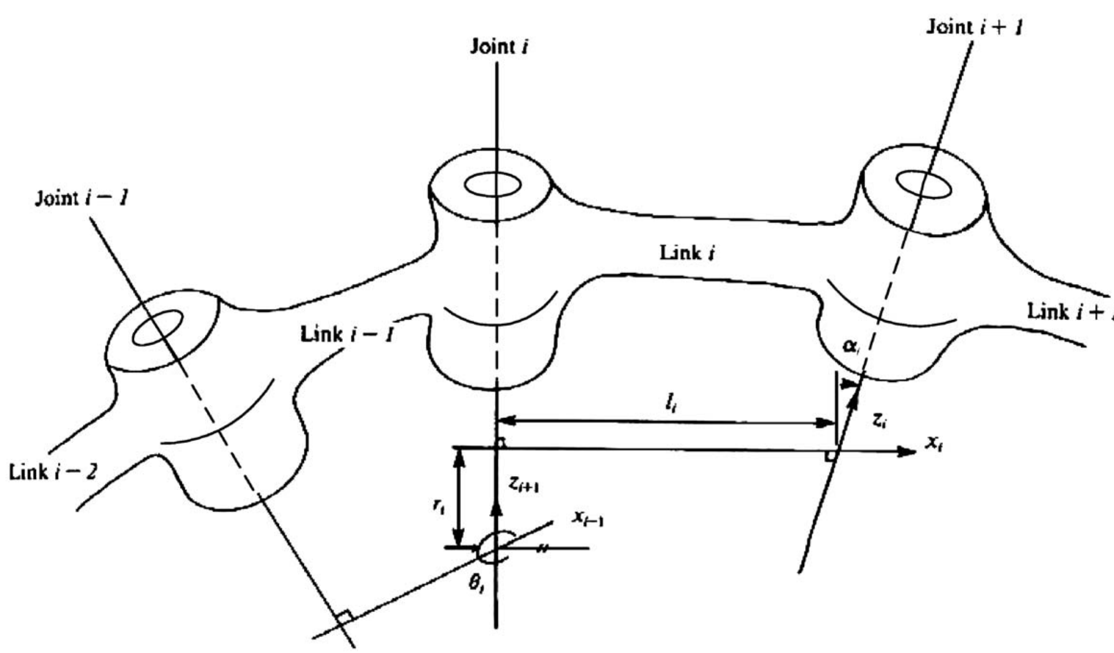

Fig. 7. Kinematic parameters of a joint (Denavit and Hartenberg, 1955).

Given errors in link parameters $\theta_i, r_i, l_i, \alpha_i$, as: $\Delta\theta_i, \Delta r_i, \Delta l_i, \Delta\alpha_i$, there will be a differential change $\mathrm{d}A_i$ between the two joint coordinate systems. Consequently, one should define the relationship between the two coordinate systems as $A_i + \mathrm{d}A_i$, where $A_i$is given in Eq. (1) and

$$\mathrm{d}A_i = \frac{\partial A_i}{\partial \theta_i}\Delta\theta_i + \frac{\partial A_i}{\partial r_i}\Delta r_i + \frac{\partial A_i}{\partial l_i}\Delta l_i + \frac{\partial A_i}{\partial \alpha_i}\Delta\theta_i. \quad (3)$$

Substituting Eq. (3) in Eq. (2) results in an error in T_N, denoted as $\mathrm{d}T_N$, given by

$$\begin{aligned} T_N + \mathrm{d}T_N &= (A_1 + \mathrm{d}A_1)*, \cdots, *(A_N + \mathrm{d}A_N) \\ &= \prod_{i=1}^{N}(A_i + \mathrm{d}A_i), \end{aligned} \quad (4)$$

where $\mathrm{d}T_N$ represents the total differential change at the end of the manipulator due to the $4N$ kinematic errors.

Observing Eq. (3), the differential change matrix, $\mathrm{d}A_i$, can be estimated using an error estimate transform matrix δA_i given by differentiating Eq. (1):

$$\delta A_i = \begin{bmatrix} 0 & -C\alpha_i\Delta\theta_i & S\alpha_i\Delta\theta_i & \Delta l_i \\ C\alpha_i\Delta\theta & 0 & -\Delta\alpha_i & l_iC\alpha_i\Delta\theta_i + S\alpha_i\Delta r_i \\ -S\alpha_i\Delta\theta & \Delta\alpha_i & 0 & -l_iS\alpha_i\Delta\theta_i + C\alpha_i\Delta r \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (5)$$

Hence $\mathrm{d}A_i$ is given by

$$\mathrm{d}A_i = A_i * \delta A_i \quad (6)$$

and

$$A_i + \mathrm{d}A_i = A_i * (I + \delta A_i), \quad (7)$$

where I is the identity matrix.

Substituting Eq. (7) in Eq. (4) and solving for $\mathrm{d}T_N$, we get

$$\mathrm{d}T_N = T_N * \delta T_N = T_N * \sum_{i=1}^{N}\left(U_{i+1}^{-1} * \delta A_i * U_{i+1}\right), \quad (8)$$

where

$$U_i = A_i * A_{i+1} *, \ldots, * A_N \quad (9)$$

and $U_{N+1} = I$, the identity matrix.

The elements δT_N in Eq. (8) define the total angular, $\delta_i^A = [\delta x^N, \delta y^N, \delta z^N]$, and translational, $\mathrm{d}_i^A = [\mathrm{d}x^N, \mathrm{d}y^N, \mathrm{d}z^N,]$, errors of the last coordinate system as

$$\delta T_N = \begin{bmatrix} 0 & -\delta z^N & \delta y^N & \mathrm{d}x^N \\ \delta z^N & 0 & -\delta x^N & \mathrm{d}y^N \\ -\delta y^N & \delta x^N & 0 & \mathrm{d}z^N \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (10)$$

Writing Eq. (8) in a matrix form one can express d^N and δ^N as

$$\begin{aligned} \mathrm{d}^N &= M_1\Delta\theta + M_2\Delta r + M_3\Delta l + M_4\Delta\alpha, \\ \delta^n &= M_2\Delta\theta + M_3\Delta\alpha. \end{aligned} \quad (11)$$

We refer the reader to Wu (1984), for a full definition of M_i.

Observing Eq. (11) one can notice that the angular error in cup alignment δ^N is affected only by the kinematic error of the angular parameters, i.e., $\Delta\theta$ and $\Delta\alpha$ which are the vector of errors in the N joints. This result implies that when dealing with cup orientation one can neglect the location parameters and their associated errors when defining the APP. On the other hand, the angular errors and angular kinematic parameters should be taken into account. Hence from now on we will not deal with the translational error and focus only on the parameters affecting the rotational error of the cup.

In order to calculate the kinematic error envelope on the end of the manipulator, a reasonable assumption would be that $\Delta\theta$ and $\Delta\alpha$ are independent N-variables, with non-zero, normal distributions with $E[\Delta\theta] = E[\Delta\alpha] = 0$, and V_θ and V_α are N by N diagonal variance matrices of $\Delta\theta$ and $\Delta\alpha$, with components: $(\sigma_{\theta 1}^2, \ldots, \sigma_{\theta N}^2)$ and $(\sigma_{\alpha 1}^2, \ldots, \sigma_{\alpha N}^2)$ on their diagonal, respectively. From this, one can find that

$$E[\delta^N] = M_2E[\Delta\theta] + M_3E[\Delta\alpha] = 0 \quad (12)$$

and the resulting variance

$$V_\delta = M_2V_\theta^{-1}M_2' + M_3V_\alpha^{-1}M_3'. \quad (13)$$

By using Eqs. (8), (12), and (13) one can define the total error and error envelope of the cup alignment as a function of the kinematic error parameters which reflect APPA error.

2.3. *Referring measurement error to the model kinematics errors*

As was presented in Section 3, cup alignment error is affected by two vectors of angles describing two of the four kinematic parameters of each joint. However, observing Figs. 3–5, one can see that errors are introduced into the model as positional errors in the acquisition of anatomical landmarks. In this section we present a methodology that transforms those errors into angular kinematic parameters.

All three angles θ_i can be expressed as (Figs. 3–5):

$$\theta_i = \tan^{-1}\left(\frac{X}{Y}\right) \quad i = 1, \ldots, 3, \quad (13\prime)$$

where X and Y are the distances determined as a function of the error in acquiring the position of the resulting anatomical landmarks. Each of these distances has a mean, μ, and a variance, σ^2. In order to relate these mean and variance values to the mean and variance of each of the θ_i one can use the *Delta Method*, given as (Mood et al., 1963):

$$E[g(X,Y))] \approx g(\mu_x,\mu_y) + \frac{1}{2}\operatorname{var}[X]\frac{\partial^2}{\partial x^2}g(x,y)\bigg|_{\mu x,\mu y} + \frac{1}{2}\operatorname{var}[Y]\frac{\partial^2}{\partial y^2}g(x,y)\bigg|_{\mu x,\mu y} + \operatorname{cov}[X,Y]\frac{\partial^2}{\partial y\partial x}g(x,y)\bigg|_{\mu x,\mu y},$$

$$\operatorname{var}[g(X,Y))] \approx \operatorname{var}[X]\left\{\frac{\partial}{\partial x}g(x,y)\bigg|_{\mu x,\mu y}\right\}^2 + \operatorname{var}[Y]\left\{\frac{\partial}{\partial y}g(x,y)\bigg|_{\mu x,\mu y}\right\} + 2\operatorname{cov}[X,Y]^2 \left\{\frac{\partial}{\partial x}g(x,y)\bigg|_{\mu x,\mu y}\cdot\frac{\partial}{\partial y}g(x,y)\bigg|_{\mu x,\mu y}\right\}. \quad (14)$$

A reasonable assumption would be that our measurements are independent, i.e.

$$\operatorname{cov}[X,Y] = 0. \quad (15)$$

Applying the *Delta Method* as given in Eq. (14), we get that

$$g(X,Y) = \arctan\left(\frac{X}{Y}\right). \quad (16)$$

Hence

$$\frac{\partial}{\partial x}g(X,Y) = \frac{1}{Y(1+X^2/Y^2)}, \quad (17)$$

$$\frac{\partial}{\partial y}g(X,Y) = \frac{-X}{Y^2(1+X^2/Y^2)}, \quad (18)$$

$$\frac{\partial^2}{\partial x^2}g(X,Y) = \frac{-2X}{Y^3(1+X^2/Y^2)^2}, \quad (19)$$

$$\frac{\partial^2}{\partial y^2}g(X,Y) = \frac{2X}{Y^3(1+X^2/Y^2)} - \frac{2X^3}{Y^5(1+X^2/Y^2)^2}, \quad (20)$$

$$\frac{\partial^2}{\partial y\partial x} = \frac{-1}{Y^2(1+X^2/Y^2)} + \frac{2X^2}{Y^4(1+X^2/Y^2)^2}. \quad (21)$$

Substituting Eqs. (16)–(21) in Eq. (14) would enable us to calculate the mean and variance per each θ_i as a function of the acquired anatomical landmarks. In order to determine the distribution of the new variables, i.e. θ_i, we simulated Eq. (13′) using a thousand X and Y data chosen from a normal distribution with $\mu_1 = 20, \sigma_1 = 15$ and $\mu_2 = 100, \sigma_2 = 10$ (as shown in Table 1). We analyzed the resulting θ, using the Jarque–Bera test. The result of the test indicated that we cannot reject the hypothesis that θ is normally distributed. The test is significant at the 5% level hence Eqs. (12) and (13) are valid. Moreover, the histogram of the resulting θ also support the results (Fig. 8).

2.4. *Abduction and version angle evaluation: the "inverse kinematics" approach*

Section 2.3 presented a mathematical method to evaluate errors in abduction and version angles as a function of anatomical measurement errors of the anatomic landmarks (pubic tubercles and the anterior superior iliac spines) that define the APP. These measurement errors result in an angular error of the anterior pelvic plane axes, and will result in cup orientation errors.

In this section we present the *inverse kinematics* solution of the kinematic skeleton given in Fig. 6, where θ_4 and θ_5 are unknown a priori and serve as parameters.

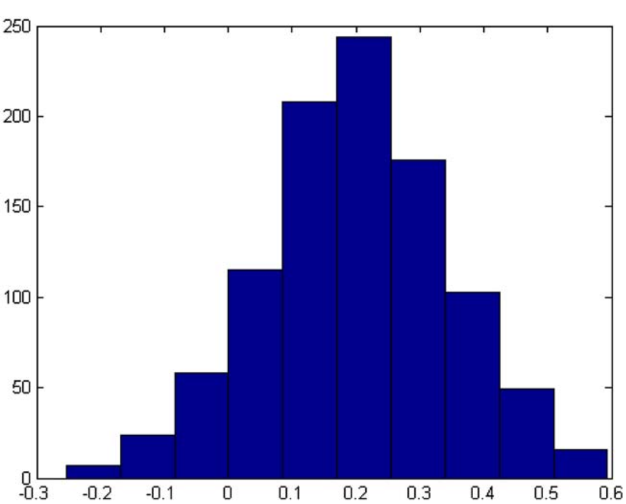

Fig. 8. Histogram of the resulting θ.

Table 1
Simulation data related to Figs. 2–4

i	μ_x	$\sigma^2{}_x$	Y	μ_y	$\sigma^2{}_y$	$\theta_i = E(\Delta\theta_i)$	Var $(\Delta\theta_i) = \sigma^2{}_{\theta,i}$
1	0	$(A/6)^2 = 11.1$	C/2	119	11.1	0	5.8
2	0	$(D/6)^2 = 11.1$	F/2	50	11.1	0	32.8
3	0	$(G/6)^2 = 11.1$	C/2	119	11.1	0	5.8

All length in millimeters and angles in degrees.

The solution provides the version and abduction angles that would orient the cup in the desired orientation, i.e., 45° and 20° abduction and version, respectively, taking into account all the measurement errors. As one can detect, in order to make use of this method in the OR one has to be able to define the measurement errors, which is a hard task to do. Yet if we are able to determine these errors then the following method would be a very powerful one as the actual cup orientation would be as planned. One may think that knowing the errors is enough in order to correct the error directly or to correct cup orientation parameters, yet this is not an easy task, as the error usually results from two independent rotations, which is hard to imagine even for a skilled person. Furthermore, in some cases, a systematic error is committed (patient positioning on the operating room table) or can be estimated (landmark correction in obese patients), yet its direct correction may not be clear. Observing Eqs. (1)–(13) one can detect that these are non-linear equations; hence having even two unknown parameters would complicate the inverse kinematic solution. The inverse kinematic solution would required, first, deriving the forward kinematic of the system as is given in Eq. (2) and then solving it for a specific orientation and location as reflected in T_N. This solution can be done manually or by using a mathematical solver.

3. Results

3.1. Error evaluation: the "forward kinematics" approach

Following is a simulation of the method presented in Sections 3 and 4. Measurement error were estimated by talking to physicians, yet a more thorough statistical research is currently been performed, so that simulations would be as accurate as possible. Parameters A, D, and G (range of errors) were estimated as 30 mm, implying a ± 15 mm error. Parameters B and E (bias) were estimated to be 10 mm (Figs. 3–5). Also C, the distance between the anterior superior iliac spines, was taken as 238 mm. Finally, F, the distance between the anterior superior iliac spines and the pelvic tubercles, was taken as 100 mm.

A summary of the simulation data and the resulting error angles $\theta_i = E(\Delta\theta_i)$ and $\mathrm{var}(\Delta\theta_i)$ by applying Eq. (14) is given in Table 1.

Applying Eqs. (1)–(13), with a version angle of 20° and abduction angle of 45°, we calculate the resulting cup orientation by solving the direct kinematics of the system as a function of all the errors given in Table 1. By solving Eqs. (12) and (13) we also get the boundary of the error, i.e., the maximum and minimum error for the abduction and version. The results are

$$\text{Mean abduction} = 49.8^\circ,$$
$$\text{Mean abduction} = 34.3^\circ,$$
$$\text{MaxError}_{\text{abduction}}(\pm 3\sigma) = \pm 15.1^\circ,$$
$$\text{MaxError}_{\text{version}}(\pm 3\sigma) = \pm 21^\circ.$$

Next we took each of the three mean angular errors $\pm 3\sigma_{\theta,i}$ and divided the range into seven sections (we present only five graphs due to space requirements). Keeping θ_2 (flexion) fixed during each iteration, we calculate the version and abduction errors caused by each combination of θ_1 and θ_3 (Figs. 9–13).

Each graph, Figs. 9–13, represents a constant value for θ_2, this value is indicated at the top of each graph. The $X-$and $Y-$axis represent the version and abduction angles resulting from $\theta_1, \ldots, \theta_3$ error combination.

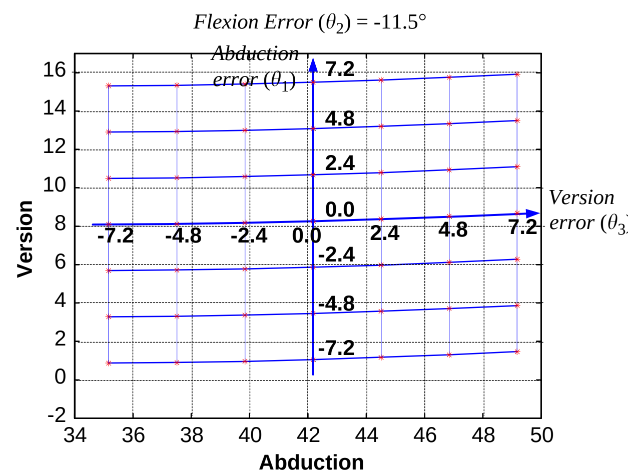

Fig. 9. Abduction and version error as a function of measurement error, flexion error = −11.5°.

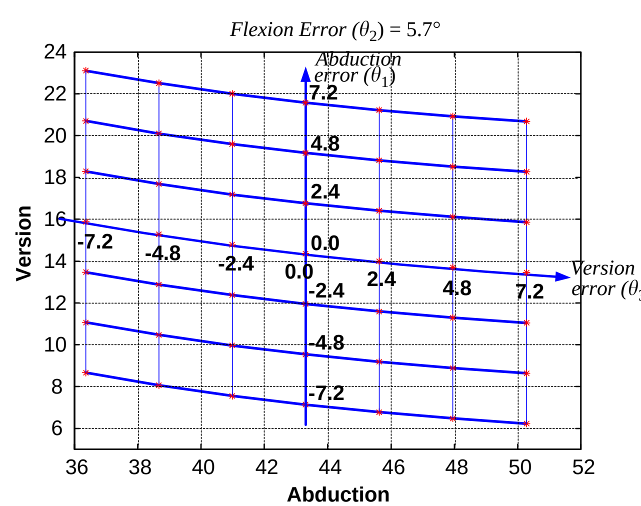

Fig. 10. Abduction and version error as a function of measurement error, flexion error = −5.7°.

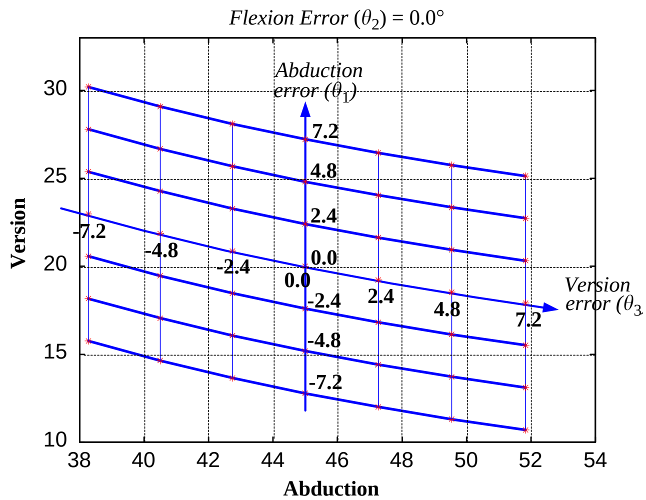

Fig. 11. Abduction and version error as a function of measurement error, flexion error = 0°.

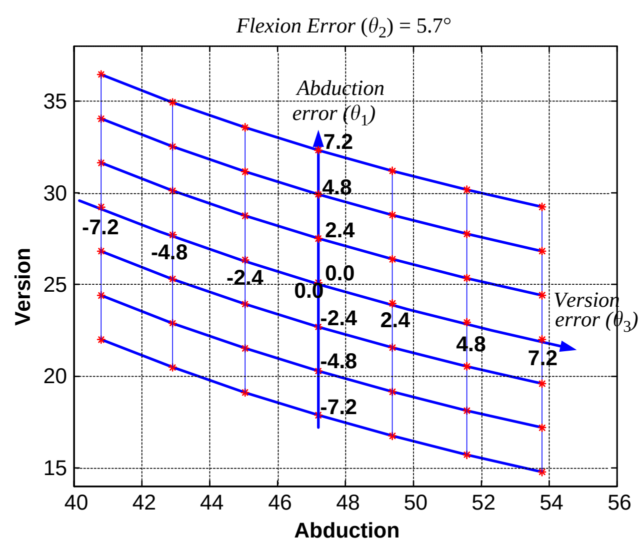

Fig. 12. Abduction and version error as a function of measurement error, flexion error = 5.7°.

These errors are given on a grid and θ_1 and θ_3 are given in parenthesis. Each horizontal line represents a constant θ_3 value, and each vertical line represents a constant θ_1 value.

3.2. *Error evaluation of the "inverse kinematics" approach*

Following are four examples of extreme errors and the resulting abduction and version error that would result in an actual cup orientation of 45° and 20° (Table 2). The solution of the inverse kinematics was derived in Maple 8 (by Waterloo Maple) using a parametric solver.

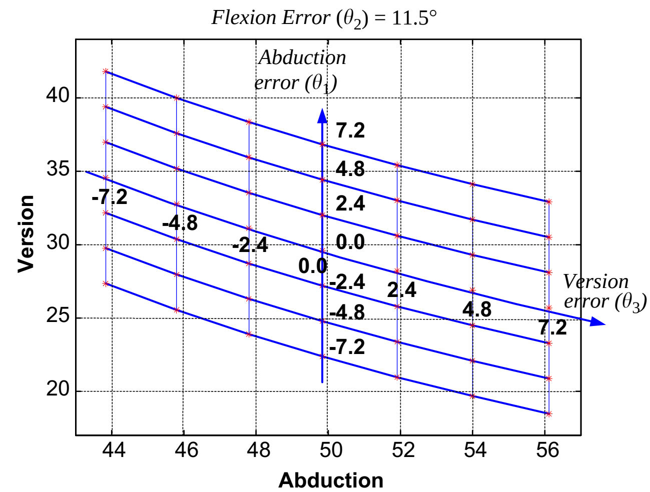

Fig. 13. Abduction and version error as a function of measurement error, flexion error = 11.5°.

Table 2
Simulation data for the inverse kinematic solution (in degrees) resulting in cup orientation of 45° abduction and 20° of version, for given errors $\theta_1, \theta_2, \theta_3$

Errors			Corrected angles	
θ_1	θ_2	θ_3	θ_4-Version	θ_5-Abduction
−7	−15	7	41.8	42.3
0	10	3	9.3	44.7
3	10	0	6.8	47.1
−7	0	−7	30.8	51.2

4. Discussion and conclusions

The APP is commonly used as an anatomical reference for cup alignment in total hip replacement procedures. Practically, all computer assisted orthopaedic surgery systems for THR which are in use today rely on the anterior pelvic plane and its axes definition, derived from two pairs of pelvic bony landmarks: anterior superior iliac spines and the pubic tubercles. While these systems strive to achieve cup alignment accuracy on the order of 1°, even a minor failure to correctly identify the anatomical landmarks can lead to much higher inaccuracies in the final cup alignment. This main contribution of the study is to derive a closed-form mathematical solution for the analysis of the effect of inaccuracies related to the determination of the location of the APP landmarks and their effect on the final cup alignment during total hip arthroplasty. Moreover, these results could also be applied to non-computerized techniques where mechanical guides are being used, as the manufacturers assume, usually wrongly, that the APP is perpendicular to the operating room table (McCollum and Gray, 1990).

Our study indicates that when orienting the cup at an abduction angle of 45° and a version angle of 20° while using erroneous landmark data (Table 1), the resulting boundary of the cup orientation errors, i.e., the maximum and minimum error for the abduction and version angles, are $49.8 \pm 15.1^\circ$ ($\pm 3\sigma$) of abduction, and $34.3 \pm 21^\circ$ of version. This generates a mean error of approximately 5° in abduction and 15° in version with a range of error of $\pm 15^\circ$ in abduction and $\pm 20^\circ$ in version.

We also presented a method to calculate corrected version and abduction angles that would result in proper cup orientation. These angles are determined by solving the inverse kinematic problem with the version and abduction angles as unknowns. This is a powerful technique when each of the three error components are known or could be estimated, say due to a systematic error that can be evaluated.

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