Several quantitative investigations focusing on lumbar vertebral dimensions have been published. Berry et al\textsuperscript{1} manually measured 30 skeletons, creating a database for implants. Nissan and Gilad\textsuperscript{8} measured the sagittal plane dimension of several anatomic structures of the vertebrae using lateral radiographs of 157 patients. Scoles et al\textsuperscript{10} studied multiple morphologic parameters in 50 normal adults. Zindrick et al\textsuperscript{16} reported on 2905 pedicle measurements taken from computed tomography data. Van Schaik et al\textsuperscript{15} also used computed tomography (CT) to provide morphologic data on anatomic structures of the vertebrae.

When the exact entry point and path orientation are needed, for example in a computerized medical robot-assisted surgery system, several vertebral dimensions are missing in the literature. In Berry et al\textsuperscript{1}, not all pedicle dimensions were measured, and there was insufficient information to calculate lateral vertebral body and disc entry. In Nissan and Gilad\textsuperscript{8} and Scoles et al\textsuperscript{10}, limited anatomic measurements were given, and no data on L2 and L4 were provided. In Zindrick et al\textsuperscript{16}, no information other than for pedicles was provided. The investigation of Van Schaik et al\textsuperscript{15} focused on the transverse process structure and dimensions. Other works studied different anatomic spine structures such as the lumbar vertebral canal\textsuperscript{4}, as well as the pedicle and vertebra body in different spinal sections, for instance, the cervical spine\textsuperscript{5,8,9} and the thoracic spine\textsuperscript{11}.

In the new systems of computer-assisted surgery, such as medical robotics, the tool trajectory’s coordinates have to be accurately defined. Moreover, for such robots to be successfully designed, the operation workspace has to be predetermined in such a manner that the required tool trajectory falls within the robot’s reachable region. Hence, workspace definition is essential for the design of the robot’s type, size, weight, and structure (mechanical strength, motors, sensors, etc.).\textsuperscript{2,3,6,7,14} Other parameters, such as forces, velocity, and accuracy, are also needed to complete the design stage\textsuperscript{12,13} but are not related to the workspace definition, which is the focus of the present investigation.

To define the operational workspace for surgical procedures on the lumbar spine, morphologic data of the lumbar spine are needed. To accomplish this, the lumbar vertebral anatomy of 55 patients was measured using data from CT, and 15 anatomic measurements were taken for each vertebra. On the basis of the measurements taken, the locations of puncture points and tool orientations for pedicle, vertebral body, and disc entry points were calculated for percutaneous as well as for open surgery. These calculations were used to define the workspace for a medical robot system for spinal surgery.

### Materials and Methods

The anatomic dimensions of 55 patients (25 women, 30 men) were measured from CT scans of L\textsubscript{1}–L\textsubscript{5}. The age distribution of the patients is given in Table 1. The mean value and variance of each dimension were calculated, and the data distribution was found.

These measurements were taken as a necessary step toward the design of the work envelope of a medical robot. Several dimensions measured in this investigation had not previously...
been clearly defined in the literature. These dimensions can also be used for implant design and modeling.

Figure 1 depicts the 15 measured vertebral dimensions.

Results

Statistical Data and Data Analysis

Tables 2 and 3 show the anatomic dimensions measured (Figure 1) and their statistics. From an analysis of the data in these tables, one can derive the location of the puncture point and tool orientation for percutaneous and open surgery performed on the spine. These procedures include, for example, pedicle screw fixation, percutaneous pedicle screw fixation, discectomy, discography, and vertebral biopsy.

To confirm that the data are normally distributed, a normal probability graph has been drawn for each dimension, and the results are given in Figure 2, A through E. In these graphs, the y axis represents the theoretical fractions of the distribution, and the ordered data are presented along the x axis. Because normally distributed data result in a straight line, Figure 2, A through E implies that all measurement data for all vertebrae are normally distributed. These results indicate that in order to include the entire population in the workspace, each measured parameter of the vertebra varies in the range $x \pm 3\sigma$, where $\bar{x}$ is its mean value (Table 2), and $\sigma$ is its standard deviation.


Table 1. Age Distribution of Patients

<table>
<thead>
<tr>
<th>Age Group</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–20 yr</td>
<td>1</td>
</tr>
<tr>
<td>20–30 yr</td>
<td>2</td>
</tr>
<tr>
<td>30–40 yr</td>
<td>5</td>
</tr>
<tr>
<td>40–50 yr</td>
<td>13</td>
</tr>
<tr>
<td>50–60 yr</td>
<td>8</td>
</tr>
<tr>
<td>60–70 yr</td>
<td>10</td>
</tr>
<tr>
<td>70–80 yr</td>
<td>10</td>
</tr>
<tr>
<td>80–90 yr</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 2. Mean Values of Measured Dimensions (mm, *degrees)

<table>
<thead>
<tr>
<th>Vertebra</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E*</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
<th>N</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>44.8</td>
<td>28.9</td>
<td>30.0</td>
<td>5.0</td>
<td>11.8</td>
<td>5.6</td>
<td>81.8</td>
<td>40.7</td>
<td>76.0</td>
<td>49.4</td>
<td>37.5</td>
<td>24.9</td>
<td>11.5</td>
<td>22.4</td>
<td>15.1</td>
</tr>
<tr>
<td>L2</td>
<td>46.9</td>
<td>29.8</td>
<td>31.5</td>
<td>4.3</td>
<td>11.0</td>
<td>7.7</td>
<td>80.4</td>
<td>39.8</td>
<td>79.1</td>
<td>48.5</td>
<td>36.9</td>
<td>25.4</td>
<td>10.8</td>
<td>21.6</td>
<td>14.8</td>
</tr>
<tr>
<td>L3</td>
<td>47.6</td>
<td>32.3</td>
<td>33.5</td>
<td>4.7</td>
<td>12.8</td>
<td>8.9</td>
<td>89.4</td>
<td>43.1</td>
<td>80.1</td>
<td>48.9</td>
<td>39.8</td>
<td>25.6</td>
<td>11.8</td>
<td>19.4</td>
<td>14.5</td>
</tr>
<tr>
<td>L4</td>
<td>47.6</td>
<td>31.7</td>
<td>32.8</td>
<td>5.3</td>
<td>14.1</td>
<td>11.4</td>
<td>90.5</td>
<td>44.1</td>
<td>79.6</td>
<td>47.2</td>
<td>43.9</td>
<td>26.5</td>
<td>11.4</td>
<td>23.2</td>
<td>14.8</td>
</tr>
<tr>
<td>L5</td>
<td>46.6</td>
<td>32.5</td>
<td>26.0</td>
<td>5.8</td>
<td>18.5</td>
<td>13.7</td>
<td>93.7</td>
<td>48.1</td>
<td>77.1</td>
<td>43.6</td>
<td>35.5</td>
<td>28.8</td>
<td>11.2</td>
<td>21.6</td>
<td>15.6</td>
</tr>
</tbody>
</table>

Table 3. Standard Deviation of Measured Dimensions in Table 2 (mm, *degrees)

<table>
<thead>
<tr>
<th>Vertebra</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E*</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
<th>N</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>2.8</td>
<td>2.3</td>
<td>3.7</td>
<td>1.1</td>
<td>1.3</td>
<td>1.3</td>
<td>0.1</td>
<td>0.0</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>L2</td>
<td>3.6</td>
<td>2.3</td>
<td>4.6</td>
<td>1.3</td>
<td>1.7</td>
<td>1.5</td>
<td>0.6</td>
<td>0.6</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>L3</td>
<td>3.7</td>
<td>1.3</td>
<td>5.7</td>
<td>1.2</td>
<td>2.2</td>
<td>1.9</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>L4</td>
<td>4.4</td>
<td>2.1</td>
<td>5.3</td>
<td>1.4</td>
<td>2.1</td>
<td>1.8</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>L5</td>
<td>5.3</td>
<td>2.1</td>
<td>5.7</td>
<td>1.4</td>
<td>3.9</td>
<td>2.2</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
</tr>
</tbody>
</table>
deviation (Table 3). In that process, 99.7% of the population is included.

By use of the data provided above, the workspace of entry points needed to reach the vertebrae for several spinal procedures was estimated. The workspace evaluation can be divided into two groups, one for percutaneous procedures and one for open surgery procedures. Workspace evaluation for spinal operational procedures is given next.

**Workspace Evaluation for Vertebral Body and Disc Puncture Points**

In Figure 3, the unknown parameters are $D$ and $\alpha$. The following definitions are used:

$$X = \left(I + Sk - \frac{B}{2}\right) - (C + Sk) \quad (1)$$
Y = \frac{G}{2} - \frac{H}{2} + \varepsilon \quad (2)

\sin(\alpha) = \frac{d_c}{l_c} \Rightarrow l_c = \frac{d_c}{\sin(\alpha)} \quad (3)

In which Sk is the skin thickness; \varepsilon, a given error; and \(d_c\), cannula diameter (I, B, C, G, and H are given in Figure 1).

By use of a trigonometric definition for tangent, the following is obtained:

\tan(\alpha) = \frac{X}{(Y + l_c)} \quad (4)

Solving (4) for \(\alpha\) by substituting (1), (2), and (3) for \(X\), \(Y\), and \(l_c\), respectively, one obtains:

\[ \alpha_1 = \tan\left(\frac{X \cdot \sqrt{Y^2 + \left(\frac{d_c}{2}\right)^2} - \frac{d_c}{2}Y}{\frac{d_c}{2}X + \sqrt{Y^2 + \left(\frac{d_c}{2}\right)^2}}\right) \quad (5) \]

\[ \alpha_2 = \tan\left(-\frac{X \cdot \sqrt{Y^2 + \left(\frac{d_c}{2}\right)^2} - \frac{d_c}{2}Y}{\frac{d_c}{2}X - \sqrt{Y^2 + \left(\frac{d_c}{2}\right)^2}}\right) \]

where \(\alpha_1\) and \(\alpha_2\) are two solution for \(\alpha\), given in Figure 3.

The second parameter needed for the determination of a puncture point is \(D\), measured from the back central line. Using Figure 3, \(D\) can be defined as:

\[ D = \frac{C + Sk}{\tan(\alpha) + \frac{G}{2} + \varepsilon + l_c} \quad (6) \]

Table 4. Workspace Evaluation for Percutaneous Cases in Figure 3

<table>
<thead>
<tr>
<th>Vertebra</th>
<th>L1</th>
<th>L2</th>
<th>L3</th>
<th>L4</th>
<th>L5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spacer length, D (mm)</td>
<td>66.2</td>
<td>65.3</td>
<td>74.6</td>
<td>74.6</td>
<td>71.2</td>
</tr>
<tr>
<td>Workspace (half) (mm)</td>
<td>20.3</td>
<td>21.3</td>
<td>31.2</td>
<td>34.0</td>
<td>31.2</td>
</tr>
<tr>
<td>Angle, (\alpha) (degrees)</td>
<td>66.5</td>
<td>68.1</td>
<td>64.8</td>
<td>65.0</td>
<td>66.7</td>
</tr>
<tr>
<td>Angular workspace (half) (degrees)</td>
<td>3.8</td>
<td>1.3</td>
<td>1.5</td>
<td>3.3</td>
<td>2.7</td>
</tr>
</tbody>
</table>

It can be seen from Equation 6 that \(D\) is most affected by \(G\), which is the transverse process length; as \(G\) increases in length, \(D\) increases. \(D\) is also affected by \(C + Sk\), representing the length of the spinous process (C) and the skin thickness (Sk); the larger the sum of the two, the longer \(D\) becomes. Equations (5) and (6) were simulated by calculating \(\alpha_1\) and \(\alpha_2\) using the statistical data provided in Table 2, with \(\varepsilon = 2\) mm, \(d_c = 1.2\) mm. The simulation results for percutaneous and open surgery procedures are given in Tables 4 and 5.

**Workspace Evaluation for Pedicle Entry Point**

From Figure 4, the following are defined:

\[ Z = (C + Sk) \cdot \tan^{-1}(E) \quad (7) \]

\[ D = \frac{K}{2} + Z \quad (8) \]

The results for Equation 8 are given in Tables 6 and 7. Next, using the length of the pedicle axis (Table 2), one can calculate the trajectory of the axis in a direction perpendicular to the spinous process. This trajectory is the screw entry distance denoted by \(dl\) (Figure 5). These data are essential in determining the workspace of the vertebra.

The screw entry distance, \(dl\), is given by:

\[ dl = A \cdot \cos(E) \quad (9) \]

Solutions for Equation 9 are shown in Table 8.
Discussion

In this investigation, the regions in which entry points and tool orientations of several spinal operations reside were defined. These regions were calculated by use of anatomic data taken from CT of the lumbar spine. These data are helpful for conventional procedures and essential for implant design, modeling, and workspace definition for robot-assisted surgery.

The workspace of surgical tools during insertion was divided into angular and displacement regions, and it differs from one vertebra to another. It has been shown that for percutaneous surgical procedures such as vertebral body biopsy and discectomy, the mean value of the puncture point is 70.4 mm from the midline, with a minimum value of 45.9 mm in L1 and a maximum value of 108.6 mm in L5. In this case, the entry angle has a mean value of 24 degrees, with a minimum of 20.7 degrees in L1 and a maximum of 27.3 degrees in L5. For open surgery cases, the mean value of the puncture point is 70.3 mm from the midline, with a minimum value of 49.8 mm in L2 and a maximum value of 97.7 mm in L4. In this case, the entry angle has a mean value of 35.4 degrees, with a minimum of 29.2 degrees in L3 and a maximum of 38.8 degrees in L5. For the pedicle entry point in percutaneous cases, a mean value of 34.2 mm from the midline was found, with a minimum value of 14.6 mm in L1 and a maximum value of 72 mm in L5. For open surgery cases, a mean value of 28.5 mm from the midline was found, with a minimum value of 14.1 mm in L1 and a maximum value of 53.1 mm in L5. In both cases, the average entry angle is 13.7 degrees, with a minimum value of 10.5 degrees and a maximum value of 22.5 degrees. This angle is dictated by the anatomy of the pedicle.

Analyzing the data given above, one can see that for angular workspace, the maximum required tilt angle is about 40 degrees, and for displacement workspace, the range varies between 46 mm and 109 mm for disc entry point, and from 14.1 mm to 53.1 mm for pedicle entry. Hence, a 60-mm displacement workspace is sufficient. As for the displacement along the pedicle axis, a range of 10 mm is sufficient.

The data given here confine the surgical tool workspace in a box of $12 \times 10 \times 10 \text{[mm}^3]$, with a tilt angle of 5 degrees to 40 degrees relative to the surface’s normal. Assuming that these data are a representative sample of the population, this box covers 99.7% of patients’ entry points and tool orientations for several spinal interventions, such as pedicle screw fixation, percutaneous pedicle screw fixation, discectomy, discography, and vertebral biopsy.

Key Points
- This study provides additional information on vertebral structure needed to calculate accurately the entry point and tool orientation in various spinal procedures.
- These statistical data are also valuable for model and implant designs and for workspace specifications for a robot-assisted surgery system.

Acknowledgment

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References


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