Instantaneous Screws of Weight-Bearing Knee: What Can the Screws Tell Us About the Knee Motion

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There are several ways to represent a given object’s motion in a 3D space having 6DOF i.e., three translations and three rotations. Some of the methods that are used are mathematical and do not provide any geometrical insight into the nature of the motion. Screw theory is a mathematical, while at the same time, geometrical method in which the 6DOF motion of an object can be represented. We describe the 6DOF motion of a weight-bearing knee by its screw parameters, that are extracted from 3D Optical Reflective motion capture data. The screw parameters which describe the transformation of the shank with respect to the thigh in each two successive frames, is represented as the instantaneous screw axis of the motion given in its Plücker line coordinate, along with its corresponding pitch and intensity values. Moreover, the Striction curve associated with the motion provides geometrical insight into the nature of the motion and its repeatability. We describe the theoretical background and demonstrate what the screw can tell us about the motion of healthy subjects’ knee.

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1 Introduction

Knee kinematics is a complex 6DOF motion composed of simultaneous rotations and translations of the thigh, about the shank. Due to the complexity of the relative motion of the two body segments, i.e., 6 DOF, few previous studies used the helical axis method to describe the motion [1–11]. Soudan et al. [1] identified the instant axis, or screw axis, as one of the widespread methods for investigating the mechanics of the human joints. Although not implemented, the researchers point out that the instantaneous screw axis of the joint changes in both place and direction during joint movement. Woltring [2] presented an analytical model to describe the effect of measurement errors from landmark position data on the definition of the finite centroid and the finite helical axis. They implemented their method while measuring the two dimensional motion of the radius and carpal bones of one subject using Vicon stereopictures. Blankenvoort et al. [4] described the finite helical axes and pitch for a flexion motion and checked the repeatability of the results by placing markers on four human knees and tracking them. Hart [5] used data from five human subjects, the, in vivo, data are presented as a path of finite helical axes for flexion of the knee from 20 to 80 deg. The authors used this presentation to assess measuring system accuracy and skin artifact influence on the measurements. In a study by Jonsson [6], the authors also used stereo-photogrammetric analysis to calculate the helical axis of the knee during flexion. They reported that in the sagittal plane, the mean helical axis was positioned close to the femoral insertion of the ligament at 80 deg of flexion and was displaced distally and anteriorly during extension. In the frontal plane, the axis had a transverse direction at 80 deg of flexion. At close to full extension, the axis was positioned distally in the lateral condyle and proximally in the medial condyle. In the horizontal plane, the helical axes ran slightly more anteromedially in the medial than in the lateral femoral condyle, but changed inclination at close to full extension and became almost parallel to the transverse axis. Bottlang [7] calculated screw displacement axes (SDAs) from its source data. The accuracy of SDA determination from such source data was evaluated for various rotational increment sizes around a revolute joint. A novel smoothing procedure, customized for this type of source data, was developed, enabling SDA detection from incremental rotations of less than 1 deg, at an accuracy appropriate for intra-operative measurement of human joint motion. Duck et al. [8] also assessed the accuracy of SDAs for describing relative motion between two moving bodies utilizing an electromagnetic tracking device. They have concluded that application of SDAs should be a useful tool for describing relative motion in joint kinematic studies. Kinzel et al. [10,11] present a mathematical formulation for extracting the instantaneous screw parameters of two moving bodies, and then distinguishes between pure rotation and pure translational motions. It then demonstrated the measurement of the relative motion of the scapula with respect to the humerus of a dog. In Ref. [12], the authors address the problem of knee pathology assessment by using screw theory to describe the knee motion and by using the screw representation of the motion as an input to a machine learning classifier. The flexions of cadaveric knees with different pathologies were tracked using an optical tracking system. The instantaneous screw parameters which describe the transformation of the tibia with respect to the femur in each two successive observation were represented as helical axes of the SDA of the motion. The resulting screws were used to train a classifier system. The system was then tested successfully with new, never-trained-before data. The classifier demonstrated a very high success rate in identifying the knee pathology.

In the current work, we elaborate and supplement on the current state of the art by mathematically extracting all the geometrical characteristics of gait from 3D measurements data, and validating their consistency. To do that, we use screw coordinates to describe the 6-DOF motion of a weight-bearing knee joint. We collect gait data of two healthy subjects who were asked to walk, barefoot, along a 10-m walkway while tracking markers are attached to their thigh and shank. Data acquisition was performed using the Vicon system at a 120Hz. We then calculate the instantaneous screw parameters that describe the momentary transformation between each two successive frames that were acquired during motion for the dominant leg (right leg for both). We use all the parameters provided by screw representation (i.e., the pitch of the screw, the intensity of the screw, the screw axis, and the distance of the screw axis to the origin of the reference system) in order to geometrically describe the relative motion between the thigh and the shank. Moreover, we also present the Striction curve of the motion as a mean of assessing data consistency. Finally, we discuss the results, their interpretation, and implementation to the analysis of other joint motion.

2 Methods

2.1 Screw Motion and Screw Parameters. In 1900, Sir Ball published, in his monumental work “A Treatise on the Theory of Screws” [13], that a screw is a straight line with which a definite magnitude termed the pitch is associated. Sir Ball’s work, alongside Chasles’ theorem “the most general rigid body displacement can be produced by a translation along a line followed (or preceded) by a rotation about that line,” laid the foundations of the Screw-Theory. This geometrical and mathematical approach for defining rigid body displacements is termed screw motion, helical

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motion, or simply a Twist. In the present work, we apply screw parameterization of motion for Biokinematics analysis of a knee. To be more specific, we use this mathematical representation of motion to describe the instantaneous, finite motion of the shank about the thigh during weight-bearing walk.

A finite screw displacement describes the relative pose of two rigid bodies or two different poses of the same body in a global reference frame [14]. As stated in Chasle’s theorem, the motion of a free body undergoing a finite transformation can be described as a combination of a translation, \( d \), parallel to a fixed axis (a line), \( A \), in space, and a rotation, \( \phi \), about \( A \). The ratio of the translation component to the rotation component is termed the pitch of the screw. We denote the \( q \)-pitch as \( q \), such that

\[
q = \frac{d}{\phi} \quad (1)
\]

The screw axis \( A \) is expressed by its Plücker line coordinates [15]. This type of motion is known in the literature by several names: helical motion, screw motion, or simply a twist defined by an axis, \( A \), and an associated \( q \)-pitch, \( q \), as is illustrated in Fig. 1. The screw parameters define a sextuple, \( s_{ij} \), given as

\[
s_{ij} = \left[ \begin{array}{c} s \\ \mathbf{s}_0 \times s \\ s \end{array} \right]^T = (L M N P Q R) \quad (2)
\]

where, \( s \), is a unit vector along the direction of the screw axis, \( s_0 \) is a position vector of a point on the screw axis, and \( q \) is the \( q \)-pitch as defined in Eq. (1). Five independent quantities, four for the axis and one for the pitch, can uniquely specify a twist. Two extreme examples are pure rotation and pure translation. For a pure rotation motion, i.e., \( q = 0 \), the screw is reduced to

\[
T = \rho s_{ij} \quad (3)
\]

For this specific case of pure rotation, the resulting screw axis, \( A \), is the axis about which the rotation occurs.

For a pure translation motion, i.e., \( q = \infty \), the screw is reduced to

\[
s_{ij} = \left[ \begin{array}{c} 0 \\ s \\ 0 \end{array} \right]^T \quad (4)
\]

and the resulting screw axis, \( A \), is a line at infinity.

The description of the displacement of a rigid body cannot be completely determined without the specification of the amplitude or intensity of the screw axis. Let \( \rho \) be the intensity of a twist, the twist is now expressed as

\[
T = \rho s_{ij} \quad (5)
\]

We refer the reader to Refs. [13–17] for a deeper discussion on screw theory and its use for representing motion of objects.

### 2.2 Experimental Setting

The most frequently used method in 3D human gait analysis involves placing markers on the skin of the analyzed segment. For the current work, 3D motion analysis was performed using an 8-camera Vicon motion analysis system (Oxford Metrics, Ltd., Oxford, UK) for kinematic data capture, at a sampling frequency of 120 Hz. Without loss of generality, seven markers (number of markers on each segment must be greater than 3) were arbitrarily placed on the thigh and seven to the shank of the subjects (Fig. 2). All subjects were then asked to walk in a self-selected speed along a 10 m walkway. While walking, the Vicon camera system continuously tracked the 3D location of the markers expressed in the system’s global coordinate system. The data exported from the Vicon Nexus contain the XYZ coordinate of the fourteen markers which were tracked.

### 2.3 Calculating the Screw Parameters of a Finite Motion

Once the XYZ coordinates of some anatomical landmarks of a moving object are obtained, the screw parameters that describe the motion of the object can be extracted. There are numerous methods reported in the literature relating to reconstruction of the motion from anatomical landmark motion data. However, the final goal is the same, given two finite configurations of a moving object, a mathematical description of that motion needs to be extracted. Finding this mathematical description is even more challenging when dealing with human motion. In such cases, the anatomical landmarks are usually placed on the skin of the subjects. Skin artifact plays an important role in the process of extracting motion data. A review on existing methods was reported recently in Ref. [18]. Without loss of generality, in our

![Fig. 1 Screw motion (helical motion), A is the screw axis, q is the pitch.](image1.png)

![Fig. 2 Markers setup on the thigh and shank (right: view from Vicon Nexus). Markers are arbitrarily placed.](image2.png)
study, we use the point cluster technique (PCT) algorithm [19,20] to extract homogeneous transformation matrices that describe the motion. The PCT is applied on a cluster of measured markers points (points) uniformly distributed on each moving segment. Each point is assigned an arbitrary scalar that can be varied at each time step and serves as a weighting factor in an optimization procedure. In this way, the contribution of a point, affected by the skin artifact, to the transformation calculation can be factored down by assigning a low weight factor to it. Consequently, the calculation of the transformation will be based mainly on markers that are less influenced by the soft tissue artifact. The eigenvalues of the inertia tensor, that is constructed using the measured points, that are less influenced by the soft tissue artifact. The eigenvalues of the inertia tensor should remain invariant. If nonrigid body movement occurs, the eigenvalues change their values during movement. An algorithm was presented in Ref. [19] to minimize the eigenvalue changes by redistributing the weight factors of each measured point at each time step. The modified set of points is then used to extract the homogeneous transformation matrices that describe the motion [19]. It is also worth mentioning that we assume that the accuracy of the raw measurements taken is as provided by Vicon motion analysis system (Oxford Metrics, Ltd., Oxford, UK). The effect of the accuracy of the measurements on the mathematical analysis of the motion is an issue to be further investigated. We then use the algorithm described in Davidson and Hunt [11, Chap. 4.6.4] to extract the screw parameters of the joint motion from these transformation matrices. Following is the highlight of Refs. [11,20].

2.3.1 Determining the Finite Twist $s_i(q)$ From $A_i$, and the Striction Curve. Given a homogeneous transformation matrix $[A_i]$ such that

$$[A_i] = \left[ \begin{array}{cccc} n_x & a_x & a_z & x_i \\
_y & a_y & a_z & y_i \\
_0 & 0 & 1 & 0 \end{array} \right]$$

(6)

The first three components of $s_i(q)$ are given as

$$([L,M,N] = ([a_x - a_y], (a_y - a_z), (a_z - a_x))$$

(7)

and the last three as

$$(P^* Q^* R^*) = \{(x_i a_y + a_z - y_i a_x - z_i a_x),\n(-x_i a_y + y_i a_z - z_i a_z),\n(-x_i a_z + y_i a_z + z_i a_z)\}$$

(8)

The combination of the screw parameters in Eqs. (7) and (8) lies on the same axis of the finite twist that is desired. We use $h^*$ to extract the last three coordinates of the screw

$$h^* = \frac{LP^* + MQ^* + NR^*}{L^2 + M^2 + N^2}$$

(9)

such that

$$(P Q R) = (P^* - h^* L Q^* - h^* M R^* - h^* N)$$

(10)

The angle of finite rotation of the screw is given by

$$\phi = a \tan 2 \left[ \frac{(a_x - a_y)^2 + (a_y - a_z)^2 + (a_z - a_x)^2}{a_x + a_y + a_z - 1} \right]^{1/2}$$

(11)

The translation component of the finite displacement that corresponds to $\phi$ is

$$s = \frac{x_i L + y_i M + z_i N}{(L^2 + M^2 + N^2)^{1/2}}$$

(12)

such that one can define the $q$-pitch (the pitch of $s_i(q)$) as

$$q = \frac{1}{2} \tan \left( \frac{1}{2} \phi \right)$$

(13)

finally the finite screw on which the required finite twist lies is

$$s(q) = (L Q N P + qL Q + qM R + qP)$$

(14)

However, the finite twist is not defined without the magnitude of the twist. This scalar multiplies all the components in $s(q)$

$$\rho = \frac{1}{2} \tan \left( \frac{1}{2} \phi \right)$$

(15)

Now, all the parameters of the twist are resolved and given in Eq. (13); the pitch, Eq. (14); the screw from which the screw axis is extracted, Eq. (15); the twist intensity.

Once the screw parameters are obtained and the screw axis is extracted from Eq. (14), the Striction curve can be constructed as the joint of two successive screw axes common normals. In other words, given that a screw axis, A, is expressed by its Plücker line coordinates [15], the Striction curve is a graphical tool that describe a set of lines. In our case, we observe a continuous motion that is discretized to 120 configurations per second (sampling frequency of 120 Hz). There is a unique screw motion (transformation) that describes the instantaneous change between two successive discreet observations. This instantaneous screw motion is defined by its instantaneous screw parameters (see Sec. 2.1), one of which is the instantaneous screw axis represented as a line by its Plücker line coordinates. Having said that, the Striction curve that is defined by the set of the instantaneous screw axis that are extracted from the measurement of the motion, would be a graphical representation of the motion.

To calculate the Striction curve, one needs to calculate the two lines common normal joining the two closest points each on a different line. The two closest points on two lines $(l_1, m_1)$ and $(l_2, m_2)$ are given by [21]

$$q_1 = f_1 + \left( \frac{f_1 l_1^2 + f_1 l_2^2 + f_1 l_1 l_2}{|l_1 \times l_2|^2} \right) l_1$$

$$q_2 = f_2 + \left( \frac{f_2 l_1^2 + f_2 l_2^2 + f_2 l_1 l_2}{|l_1 \times l_2|^2} \right) l_2$$

(16)

where $f_i$ is a pedal point on each line, given by

$$f_i = \frac{1/2}{|l_i|}$$

(17)

The Striction curve is then constructed as the curve connecting all points $q_i$.

2.4 Gait Cycle (GC). It is common to analyze a single gait cycle assuming there is a consistency between individuals’ successive gaits. A gait cycle is defined from heel strike to heel strike.
of the same limb (i.e., 100% of gait cycle). A gait cycle is composed of two major phases: stance phase, (about 60% of gait cycle) during which the foot is in contact with the ground, and swing phase (about 40% of gait cycle), during which the foot is in the air for limb advancement (Fig. 3). The stance phase can be further subdivided into initial contact 0–2%; load response 0–10%; midstance 10–30%; terminal stance 30–50%; preswing 50–60%; terminal contact 60%—toe off [22]. Also the swing phase can be further subdivided to initial swing 60–73%, midswing 73–87%, and terminal swing 87–100%. Note that a gait cycle is defined as a sequence of motions executed from loading response to terminal swing, total of 100% gait cycle [23]. We analyze the data extracted from the gait measurement according to this gait cycle.

We used Matlab (by MathWorks) to perform data analysis. Once data were imported to Matlab, the XYZ trajectory of the markers is plotted to visualize the consistency and continuity of the motion acquired (Fig. 4). Following data verification, the screw parameters of the gait were calculated according to the pipeline described in Secs. 2.3 and 2.4.

3 Results

We present in this paper, selected graphs extracted from two subjects’ dominant leg, gait data. The screw parameters that were obtained are the screw instantaneous Axis, $A$ (Figs. 5 and 6), $q$-pitch, $q$, (Figs. 7 and 8), screw intensity, $\rho$, (Figs. 9 and 10), and Striction curve (Figs. 5 and 6). Also given in Fig. 11 is the knee flexion/extension angles extracted from the Vicon Polygon report.

A total of fourteen markers were used to track the lower limb kinematics. Seven markers were placed on the thigh and seven on the shank. Given in Fig. 4 are the trajectories of all markers in XYZ coordinates. As can be noted, the walkway is oriented along the $Y$ axis of the measurement systems.

4 Discussion

Without loss of generality, we focus our discussion of the screw parameters given in example 1 (subject 1). Looking at Fig. 7 (upper graph), one can detect that the pitch values are cyclical, starting with the heel strike (0% gait), the pitch value of the motion is fairly large meaning that the translational part of the motion is more dominant than the rotational (see Eq. (1)). This value decreases until the middle of the terminal stance...
(approximately 39%GC), meaning that the nature of the motion becomes more dominated by the rotational motion. From the middle of the terminal stance (39%GC) toward the end of the initial swing (frame 180–68%GC), the pitch value plateaus, with pitch values close to zero meaning that the nature of the shank motion about the thigh is close to a pure rotational motion (Eq. (3)). During mid swing the pitch value drops again, reflecting a decrease in the domination of the rotation and increase of translation. This motion lasts till the end of the mid swing (frame 210 89%GC). At terminal swing, the pitch value decreases again, indicating that the rotation becomes more dominant again. This pattern repeats in the following gaits.

Looking at Fig. 9, it is interesting to note that the screw intensity is relatively low throughout the gait, having a high peak during the initial part of the gait and a very high peak during the mid swing part of the gait cycle (67% GC). In this phase of the motion (67% GC), the shank is being flexed (Fig. 3). This phase of the gait cycle is known for its high acceleration of the shank to advance the limb [24, 25].

We plot in Fig. 7 (lower plot) the distance of the screw axis about the origin of the reference coordinate system, which is located on the thigh. Note that the distance is cyclic, indicating
what is known by now, that the screw axis of the knee is indeed an instantaneous one and not fixed during gait. That is, the knee joint in a revolute joint. The screw axis distance is increasing toward the end of the terminal stance and the beginning of the heel off phase. In this phase, the knee is fully extended (38% GC). The screw axis distance is decreasing during the initial swing phase and beginning of the midswing (67% GC). In this phase, the knee is in maximum flexion, so that the thigh and shank are closer to each other. This finding corresponds also to the knee angle changes during gait, as reported in Whittle [23]. As can be noted in Fig. 12, the knee should be in about 4 deg of extension around 35% of the GC and a maximal flexion angle of about 60 deg around 70% of GC. This observation is supported by the measured knee flexion/extension angle which is plotted, for both examples, in Fig. 11.

Finally, we refer the reader to Fig. 5, where both the screw axes are plotted alongside the Striction curve describing the motion. As demonstrated in Sec. 2.3.1, the Striction curve is constructed as the joining of two successive screw axes common normals. It is important to note that the resulting graph is a closed curve and that the two gait cycles’ Striction curves overlap. As described in Sec. 2.3.1, the Striction curve is constructed given the set of screw axes (lines) that are part of the screw parameters that are calculated. The fact that the calculations resulted in a closed curve mean that the thigh and shank relative motion is cyclic, as is clinically expected for a healthy subject. Moreover, the fact that the graphs overlap indicate that the relative motion is repeatable from a kinematics point of view. Also note the same behavior in Fig. 6 where three gait cycles overlap (subject 2). These findings can also reflect on the accuracy and precision of the date obtained by the measurement system; however, this matter needs to be further investigated.

We would like to point out that the same analysis of the gait and observations can be extracted from example 2 (subject 2). Similar patterns can be detected in subject 2 gait.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Subject 1 frames (%gait)</th>
<th>Subject 2 frames (%gait)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full gait</td>
<td>90–223 (100)</td>
<td>30–160 (100)</td>
</tr>
<tr>
<td>Stands</td>
<td>90–166 (58)</td>
<td>30–105 (59)</td>
</tr>
<tr>
<td>Swing</td>
<td>167–223 (42)</td>
<td>106–160 (41)</td>
</tr>
<tr>
<td>Initial-mid swing</td>
<td>167–205 (59–86)</td>
<td>106–147 (59–89)</td>
</tr>
<tr>
<td>Terminal swing</td>
<td>206–223 (86–100)</td>
<td>148–160 (89–100)</td>
</tr>
</tbody>
</table>
5 Conclusions

In this paper, we presented a Bio–Kinematics approach for gait analysis. We used screw parameterization of motion to represent the kinematics of the knee joint while weight bearing. The screw parameters were extracted from the set of homogeneous transformation matrices that were obtained using the PCT. The screw parameters include the Plücker coordinates of the screw axes, the associated pitch value of the screw, the distance of the screw axis about the origin of the reference frame, the screw intensity, and finally the Striction curve, which represents the motion. As was demonstrated in this paper, the screw parameters do not just provide a mathematical representation of the motion but, more importantly, they provide a geometrical–physical description of the motion. That is to say, the screw parameters can be visually presented and interpreted as quantitative parameters of the motion. For example: The instantaneous screw axes indicate where, in space, the actual axes, about which the motion is generated is during gait. This axis can be located anteriorly or posteriorly to the knee anatomical axis as a function of the gait phase. The screw intensity corresponds with the relative acceleration of the two body segments, i.e., the thigh and shank. In our case, it dramatically increases during the initial and mid swing part of the gait cycle. Finally, the screw pitch indicates the nature of the motion between the thigh and shank, whether it is a rotational, translational nature or a combination of the two. For the knee motion, the pitch values (as all parameters are) are cyclical and changes from a translation in nature to a rotation and back again. It is easy to follow the pitch values graphically and see what the dominant motion is during the gait cycle. The Striction curve provides a graphical representation of the motion. It indicates the repeatability and cyclical nature of the motion. All these parameters, i.e., screw axis, pitch, and intensity also have an engineering importance. For example, a medical-device designer can use these parameters to better design an external fixator device taking into account the shifting of the axis of rotation, the rotational portion and translational portion of the joint motion during gait. An implant designer may use these geometrical parameters to design more anatomical like implants, ones that would, not only physically but also kinematically, mimic the biological knee motion. This could be simulated during the design phase and then implemented mechanically.

There is, however, few limitations to this study. First and foremost, only two subjects were represented. While the knee kinematics can be represented in an invariant fashion using a screw axis, ultimately the data need to be represented in anatomical coordinate frames to be used in a clinical setting. Although we present in Ref. [12], a partial solution to that during cadaveric study, this issue needs to be further addressed in future studies on human motion. Furthermore, it is known that skin mounted markers introduce measurements error known also as skin artifact. This issue draws the attention of the biomechanics community (and this author too). Several mathematical methods are being developed in order minimize the skin artifact; however, this still remains a source of error [18]. To overcome skin artifact, one can use more invasive methods to track the limb motion, e.g., by using invasive pins on which the markers are attached or by using bipolar fluoroscopy. Both methods are invasive and can have an effect on the subject motion, and also imposed some ethical issues. It is worth mentioning that these methods are often used during cadaveric studies; however, the measured motion does not reflect on real human motion mainly due to the lack of muscle activity and proper iabela forces. In light of that it is expected that the results presented in this study (as in any other similar study) might be affected by the skin artifact, we did not quantify nor compared the effect it has on the method reported in this paper (i.e., screw representation) or on the effect it has on other methods being used. This issue could be addressed in future studies.

Nevertheless, it is worth mentioning that motion tracking technology is constantly improving in accuracy and reliability. We believe that current characteristics of most systems have reached a level where accuracy is within the tolerated error for biomechanics analysis. It is due to this relatively new technology and modern gait-labs that such research and description of motion is enabled. Moreover, the tools and methodology presented in this paper can be applied to other joints and body segments, whether they are biological or not.

References